

# Landau-Zener Tunnelling in Waveguide Arrays

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Landau-Zener tunnelling is discussed in connection with optical waveguide arrays. Light injected in a specific band of the Bloch spectrum in the propagation constant can be transmitted to another band, changing its physical properties. This is achieved using two waveguide arrays with different refractive indices, which amounts to consider a Schrödinger equation in a periodic potential with a step. The step causes wave “acceleration” and thus induces Landau-Zener tunnelling. The region of physical parameters where this phenomenon can occur is analytically determined and a realistic experimental setup is suggested. Its application could allow the realization of light filters.

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When a quantum system is subject to an external force, a non-adiabatic crossing of energy levels can occur. This phenomenon is known as *Landau-Zener tunnelling* [1, 2] and some of its recent observations are for Josephson junctions [3] and optical effective two-level systems [4]. On the other hand, the problem of quantum motion in a periodic potential was solved already in the 1920’s (see e.g. [5]) and gives rise to band spectra and Bloch states. Nowadays, the observation of Landau-Zener tunnelling between Bloch waves is at the frontiers of research in Bose-Einstein condensates (BEC) in optical lattices [6, 7, 8, 9, 10, 11]. The external forcing, responsible for Landau-Zener tunnelling, is created by either placing the BEC in a gravitational potential [6] or accelerating the optical lattice itself [7].

In this Letter we propose a new way of generating Landau-Zener tunnelling. We consider waveguide arrays [12], where the periodic potential of the Schrödinger equation is provided by the spatial oscillation of the refractive index in the transversal direction. Tunnelling is caused by combining two waveguide arrays with different refractive indices (see Fig.1). As we will see, this corresponds to creating a *step* in a periodic potential.

For arrays of coupled waveguides, the longitudinal direction  $z$  (see Fig.1), along which the refractive index is constant, plays the role of “time” in the stationary regime. The refractive index varies only along the transversal direction  $x$ , which represents space in a  $1+1$  (space-“time”) dimensional picture. Various linear and nonlinear phenomena have been observed in waveguide arrays: discrete spatial optical solitons [13], diffraction management [14], excitation of Bloch modes [15], generation of multiband optical breathers [16] and of single band-gap solitons [17], anomalous band-gap transmission regimes [18]. Fast progress in discovering various nonlinear effects in waveguide arrays has been possible due to the introduction of the *tight-binding* approxima-

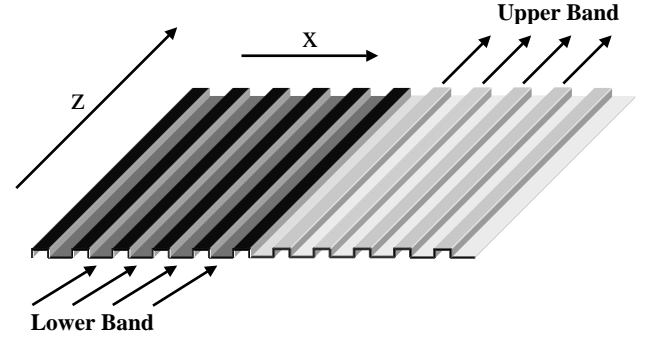


FIG. 1: Schematic picture of the combination of two waveguide arrays of the same spatial period but with different refractive indices (shown by the different grey levels). Choosing appropriately [however, see formula (9) below] the difference in refractive indices the intensity in the lower band mode is almost completely transferred to the upper band mode due to Landau-Zener tunnelling.

tion [19, 20, 21], which reduces the nonlinear Schrödinger equation to the *discrete* nonlinear Schrödinger equation [12].

However, such a reduction eliminates the rich band structure of the periodic medium and only a single Bloch band is left. On the contrary, we want to maintain the band structure and, hence, study transitions between the bands. Indeed, as we will see below, the coupling of two waveguides with different refractive indices introduces a step in the periodic potential and, consequently, Landau-Zener tunnelling. In the following we will study only transitions between the first two bands, denoting them *upper* and *lower* band. In particular, we propose to inject light into the left waveguide array with a given angle in order to populate the lower Bloch band, and retrieve it

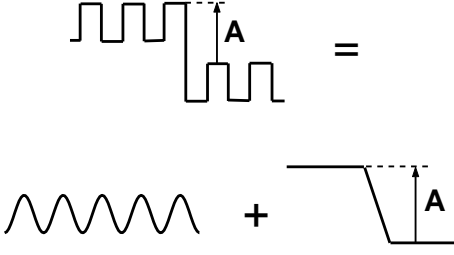


FIG. 2: Approximation of the periodic potential with a step used in the Schrödinger equation describing the waveguide arrays with different refractive indices.

from the right waveguide array. Below, we derive analytically the step size bounds inside which most of the intensity of the lower band mode is transferred to the upper band mode, creating also a spatial separation between lower and upper band light. We demonstrate this effect by performing numerical simulations of the Schrödinger equation in a periodic potential with a step.

The waveguide array refractive index profile in the transversal direction  $x$ , which has a periodic rectangular shape, is approximated by a harmonic potential, while the step in the refractive index is substituted by a slope  $-\alpha$  of height  $A$  (see Fig. 2). The functional form of the harmonic potential is selected in order to keep the ground state at zero energy, as done in the case of BEC [7]. Such an approximation allows a simple analytical treatment of the problem. Thus, in the linear regime, the adimensionalized Schrödinger equation of the optical system can be written as follows (see e.g. Ref. [22])

$$i\frac{\partial\Psi}{\partial z} + \frac{1}{2}\frac{\partial^2\Psi}{\partial x^2} + \left(V(x) + 2w\sin^2 x\right)\Psi = 0, \quad (1)$$

where  $\Psi$  stands for the complex envelope of the electric field and  $w$  is the height of the harmonic potential. Moreover,

$$\begin{aligned} V(x) &= 0 \quad \text{for } x < 0; & V(x) &= -A \quad \text{for } x > A/\alpha \\ \text{and} & & V(x) &= -\alpha x \quad \text{for } 0 < x < A/\alpha. \end{aligned} \quad (2)$$

Via the simple transformation  $\Psi \rightarrow \Psi \exp[izV(x)]$ , wave equation (1) gets a well known form (see Refs. [9, 10, 11]):

$$i\frac{\partial\Psi}{\partial z} + \frac{1}{2}\left(\frac{\partial}{\partial x} - i\alpha z\right)^2 \Psi + 2w\sin^2(x)\Psi = 0 \quad (3)$$

where  $\alpha$  plays the role of acceleration in the Landau-Zener phenomenon. However, we should keep in mind that acceleration takes place only within the step region  $0 < x < A/\alpha$ , unlike the previously considered cases of BEC's, where the whole condensate is accelerated (see e.g. Ref. [11]). Using a two mode approximation the wave function  $\Psi$  is written as follows

$$\Psi = \left[a(z)e^{iKx} + b(z)e^{i(K-2)x}\right]. \quad (4)$$

With our conventions the zone-boundary mode wavenumber is 1. It should be mentioned that the two mode approximation works better just in the vicinity of zone boundaries ( $K \rightarrow 1$ ), exactly where Landau-Zener tunnelling takes place. Following Ref. [9], we substitute Expr. (4) into the wave Eq. (3). Assuming  $K = 1$  and removing a common phase dependence in  $a(z)$  and  $b(z)$ , we get the Landau-Zener model [1, 2] in its original form

$$\begin{aligned} i\frac{\partial a}{\partial z} &= -\alpha z a + \frac{w}{2}b \\ i\frac{\partial b}{\partial z} &= \alpha z b + \frac{w}{2}a. \end{aligned} \quad (5)$$

Thus, according to Landau-Zener's result, tunnelling from the lower zone-boundary mode to the upper band takes place with the following rate

$$r = \exp\left[-\frac{\pi w^2}{4\alpha}\right]. \quad (6)$$

We consider cases in which the acceleration constant  $\alpha$  is large. Thus, the tunnelling rate is close to one, meaning that almost all light intensity approaching the step is transferred to the upper band. This is confirmed by numerical simulations.

Outside the step, where acceleration is absent, one can write down the wave-functions and the dispersion relations in simple approximate form (see e.g. [5]).

$$\Psi_{\pm} = \left[\frac{2\kappa \pm \sqrt{w^2 + 4\kappa^2}}{w}e^{-ix} - e^{ix}\right]e^{i(\beta_{\pm}z + \kappa x)} \quad (7)$$

$$\beta_{\pm}(\kappa) = \frac{\pm\sqrt{w^2 + 4\kappa^2} - 1 - \kappa^2}{2}, \quad (8)$$

where  $\kappa = K - 1$  is the wavenumber detuning from the zone-boundary and the  $+$ ( $-$ ) sign indicates the upper (lower) band.  $\beta$  is the dimensionless propagation constant. The dispersion relations (8) for the two bands are schematically shown in Fig. 3. The picture is similar to the one observed in experiments [17]. Let us remark that at the zone-boundary ( $\kappa \rightarrow 0$ ), the amplitudes of the upper and lower band modes are  $|\Psi_+| = 2\sin x$  and  $|\Psi_-| = 2\cos x$ , respectively. This means that the light intensity in the lower band is concentrated inbetween the waveguides centers, while, in the upper band, intensity is concentrated on waveguides centers. This property is a clear experimental indication whether the wave is in the lower or upper band.

The experimental setup could be as follows. One should inject a lower band mode with non zero but small relative wavenumber  $\kappa$ . This is accomplished by choosing for the light beam a direction forming an angle  $\theta$  with respect to the  $z$  direction such that  $\tan\theta = \kappa$ . Hence, the wave front will move towards the step. An analysis of the dependence of the tunnelling mechanism on the physical parameters appearing in Fig. 3 shows that the transition

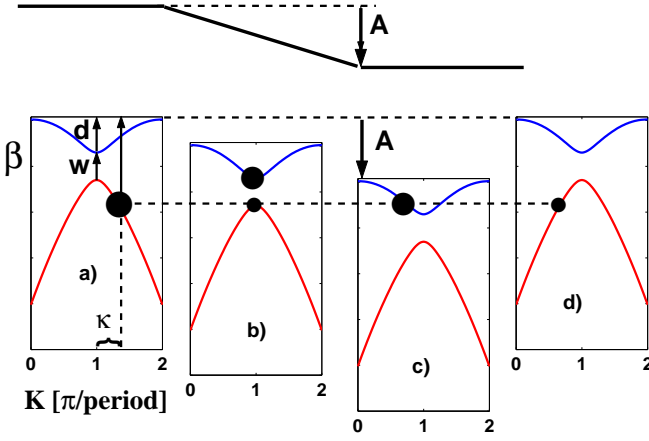


FIG. 3: Schematic band-gap structure and picture of the Landau-Zener tunnelling process.  $w$  is the gap between the bands (the height of the period potential),  $d$  is the width of the upper band and  $\kappa$  is the initial detuning of the lower band mode wavenumber from zone-boundary.  $\beta$  is the dimensionless propagation constant (see formula 8).  $A$  is the height of the step. Initially, light is injected in the left array, populating the lower band mode (a). Going across the step  $\kappa$  decreases, reaching zero as the mode approaches the zone-boundary causing Landau-Zener tunnelling (b). After tunnelling, most of the light intensity is transferred to the upper band mode, whose wavenumber decreases until the end of the step is reached. This is the light we observe in the right array (c). A smaller light intensity remains in the lower band and is observed in the left array (d).

to the upper band mode verifies only if the refraction index step  $A$  fulfills the following inequalities

$$\Delta\beta + w < A < \Delta\beta + w + d, \quad (9)$$

where  $\Delta\beta = \beta_-(0) - \beta_-(\kappa)$  is the variation of the propagation constant between the initial state and zone-boundary and  $d$  is the width of the upper band.

Let us try to justify this result by commenting at the same time the results of some numerical simulations. These are performed by fixing  $w = 0.5$ ,  $\kappa = 0.2$  and varying the step size  $A$ . Waveguide centers are placed every period, with the first waveguide at half a period from the left boundary. The refractive index step is placed in the middle of the array. If the refractive index step  $A$  is within the above limits (9), the lower band mode cannot overcome the step and when it reaches the zone-boundary Landau-Zener tunnelling to the upper band occurs. Light is partially transmitted to the right in the upper band and reflected to the left in the lower band. This regime is demonstrated in Fig. 4, which evidences, even visually, the fact that light in the left array is concentrated in-between waveguides, while it lies at waveguide centers on the right array [see also the form of wavefunctions (7)]. If the step size is higher than the upper bound of Exp. (9), the lower band mode cannot overcome the step

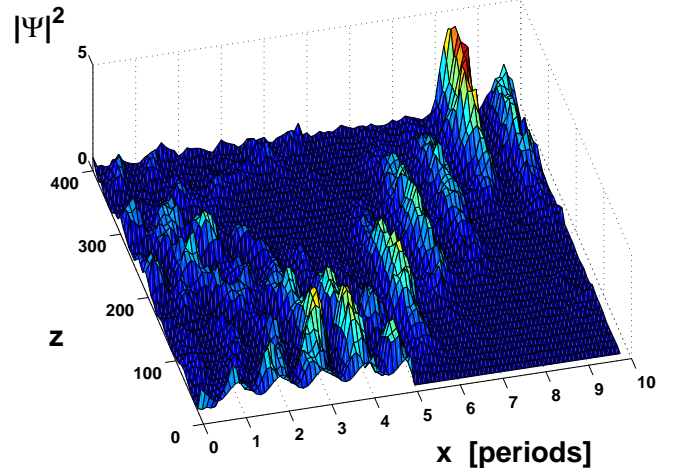


FIG. 4: Landau-Zener tunnelling from the lower to the upper band. Step size  $A = 0.7$  is taken within the limits of Exp. (9). Step is placed at  $x = 5$  and lower band mode is injected into the first five periods.

and total reflection takes place (see the upper graph of Fig. 5). On the other hand, if

$$A < \Delta\beta \quad (10)$$

the lower band mode is able to overcome the step and to penetrate to the right side without tunnelling (see the lower graph of Fig. 5). Indeed, wave intensity is now concentrated in-between the waveguides and one can conclude that only lower band modes are present in the array.

Finally, if the step is located within the following limits

$$\Delta\beta < A < \Delta\beta + w, \quad (11)$$

the wave on the left array has no counterpart on the left with the same propagation constant, thus, no stationary penetration of the light through the step is possible, more or less like in the upper Fig. 5.

Concluding, a novel type of linear optical tunnelling effect is discovered. This is discussed in connection with waveguide arrays, which have a spatially oscillating refractive index. We explain this effect resorting to Landau-Zener model, which is commonly used for accelerated quantum-mechanical two-level systems. In our case, tunnelling between different bands takes place while a wave passes through a step in the refractive index. In waveguide arrays, Bloch bands in the propagation constant (light wavenumber) play the role of energy bands in quantum mechanics. Numerical simulations show that, if certain limits in the refractive index step are respected, a spatial separation of light in different bands can be achieved. More interesting for applications is, perhaps, the use of this mechanism to build

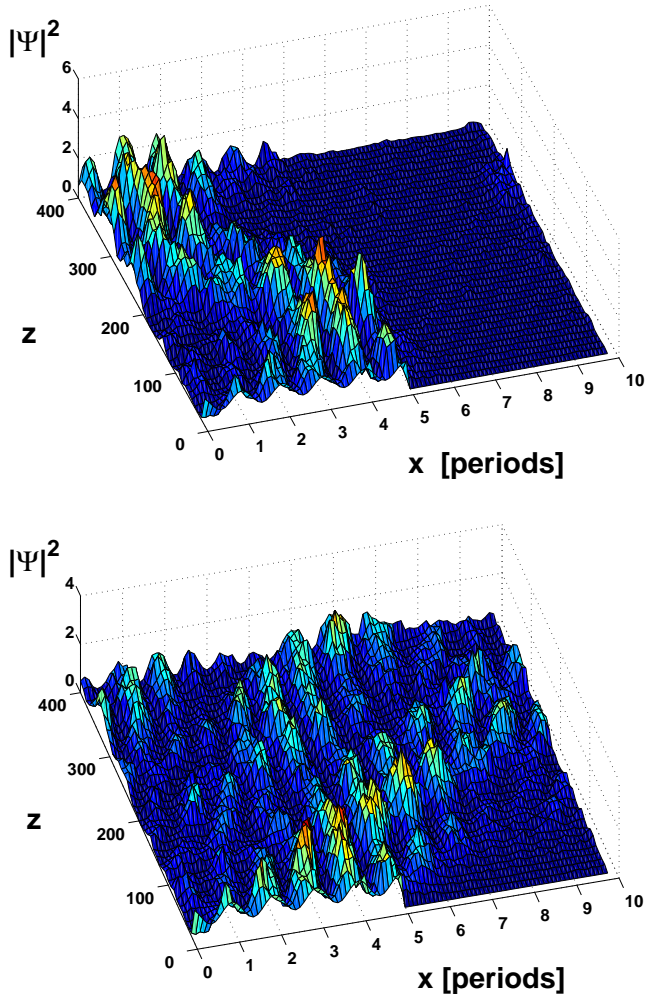


FIG. 5: Upper graph: Total reflection from the step. Step size  $A = 1$  is taken higher than the upper bound of Exp. (9). Lower graph: Penetration through the step without tunnelling. Step size  $A = 0.07$  is taken within the step values form the inequality (10).

*light wavenumber filters.* Indeed, by appropriately choosing the physical parameters, it could be possible to shift the central wavenumber of a light beam and to reduce the spread in both frequency or wavenumber. The inclusion of a weak nonlinearity does not qualitatively alter the picture, while strongly nonlinear cases need a separate treatment. In particular, the scattering process of optical gap solitons with the step could be of special interest. Moreover, it should be observed that the effect discussed in this Letter is generic for systems with periodic potential and that applications in different fields could be found. For instance, the analysis of a similar process in Bose-Einstein condensates is in progress.

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- [1] L.D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932).
- [2] G. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932).
- [3] K. Mullen, E. Ben-Jacob, Y. Gefen, and Z. Schuss, Phys. Rev. Lett., **62**, 2543 (1989); S. Fishman, K. Mullen, E. Ben-Jacob, Phys. Rev. A, **42**, 5181 (1990).
- [4] D. Bouwmeester, N. H. Dekker, F. E. v. Dorsselaer, C. A. Schrama, P.M. Visser, and J. P. Woerdman, Phys. Rev. A, **51**, 646 (1995).
- [5] C. Kittel, *Introduction to solid state physics*, 5th edition, Wiley (1976), Chapt. 7.
- [6] B.P. Anderson and M. Kasevich, Science, **282**, 1686 (1998).
- [7] M. Cristiani, O. Morsch, J.H. Müller, D. Ciampini and E. Arimondo, Phys. Rev. A, **65**, 063612 (2002).
- [8] Qian Niu, Xian-Geng Zhao, G. A. Georgakis, and M. G. Raizen, Phys. Rev. Lett., **76**, 4504 (1996).
- [9] B. Wu, Q. Niu, Phys. Rev. A, **61**, 023402 (2000).
- [10] J. Liu, L. Fu, B.-Y. Ou, Sh.-G. Chen, D.-I. Choi, B. Wu, Q. Niu, Phys. Rev. A, **66**, 023404 (2002).
- [11] M. Jona-Lasinio, O. Morsch, M. Cristiani, N. Malossi, J. H. Müller, E. Courtade, M. Anderlini, and E. Arimondo, Phys. Rev. Lett., **91**, 230406 (2003).
- [12] D.N. Christodoulides, R.I. Joseph, Opt. Lett., **13**, 794, (1988); Phys. Rev. Lett., **62**, 1746, (1989).
- [13] H. S. Eisenberg, Y. Silberberg, R. Morandotti, A.R. Boyd and J. S. Aitchison, Phys. Rev. Lett., **81**, 3383 (1998).
- [14] H. S. Eisenberg, Y. Silberberg, R. Morandotti and J. S. Aitchison, Phys. Rev. Lett., **85**, 1863 (2000).
- [15] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti and J. S. Aitchison, Phys. Rev. Lett., **90**, 053902 (2003).
- [16] D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Phys. Rev. Lett., **90**, 253902 (2003).
- [17] D. Mandelik, R. Morandotti, J. S. Aitchison, and Y. Silberberg, Phys. Rev. Lett., **92**, 093904 (2004).
- [18] R. Khomeriki, Phys. Rev. Lett. **92**, 063905 (2004).
- [19] A. A. Sukhorukov, Yu. S. Kivshar, O. Bang, and C. M. Soukoulis, Phys. Rev. E, **63**, 016615 (2001).
- [20] A. Smerzi and A. Trombettoni, Phys. Rev. A, **68**, 023613 (2003).
- [21] M. Öster, M. Johansson, A. Eriksson, Phys. Rev. E, **67**, 056606, (2003).
- [22] A.A. Sukhorukov, Yu.S. Kivshar, J. Opt. Soc. Am. B, **19**, 772, (2002).